Uludağ University Journal of The Faculty of Engineering, Volume 20, Number 2, 2015 RESEARCH
doi: 10.17482/uujfe.87936

SCATTERING OF PLANE WAVES BY A HALF-PLANE AT THE INTERFACE OF TWO ISOREFRACTIVE MEDIA

Hüsnü Deniz BAŞDEMİR *

Abstract: Scattering of plane waves by a half-plane at the boundary of two isorefractive media is analyzed. The E-polarization case is taking into account. The results were analyzed numerically and compared with a similar previous study. The results are consistent with each other.

Keywords: Diffraction, Isorefractive media, Scattering

İki İsorefractive Ortamın Arayüzündeki Yarım Düzlemden Düzlemsel Dalgaların Saçılması

Özet: İki isorefractive ortamın sınırındaki yarım düzlemden düzlemsel dalgaların saçılması analiz edilmiştir. E-polarizasyon durumu dikkate alınmıştır. Sonuçlar sayısal olarak analiz edilmiş ve bir önceki benzer çalışmayla karşılaştırılmıştır. Sonuçlar bir birleri ile tutarlıdır.

Anahtar Kelimeler: Kırınım, Isorefractive ortamlar, Saçılma

1. INTRODUCTION

It is known that a wave propagates through unobstructed medium until it encounters an obstacle. When the wave hits an obstacle, reflects from uniform part of the obstacle, diffract from its discontinuities or transmits passes through. The refraction process occurs between the interfaces of two media as different from others. The mediums have different dielectric constants. The amount of the energy of wave's reflection and transmission is determined by the reflection and transmission coefficients (Balanis, 1989). To be a special case of dielectric media, isorefractive media have same wave numbers but different intrinsic impedances and also angle of reflection is equal to the angle of incidence (Uslenghi, 2004b).

There are lots of studies in the literature related with the isorefractive media (Uslenghi, 2004a-2004b, Daniele and Uslenghi, 1999). In generally, these studies focus on the division of isorefractive media for different angles. It means that the borders of the mediums in the space are taken different than each other. Unlike others, there are only one study is performed by a half-plane at the interface between isorefractive media by Uslenghi (2013). The present study takes into account the same problem but with a different approach. The application of the isorefractive media arise in the improvement of the prompt aperture efficiency of impulse radiating antennas (IRA's) (Tyo and others, 1998). Also, an analytical solution of the radiation from an axisymmetric spheroidal slot antenna with different types of confocal isorefractive coatings was obtained (Hamid and Cooray, 2007). These studies can be given as application examples of isorefractive media.

* Çankaya University Electronic and Communication Department, Etimesgut, 06790 Ankara.
Corresponding Author: H.D. Başdemir (basdemir@cankaya.edu.tr)

In this study, scattering of plane waves by a perfectly conducting half-plane at the interface between two isorefractive media is analyzed. The geometric optics (GO) and diffracted fields are also obtained. The effect of the diffracted fields on the scattered fields is observed. The method which is used in this present work is based on the closed form series solution by defining the boundary conditions in terms of soft and hard surfaces (Umul and Yalçın, 2010). The soft half-plane (E-polarization case) is considered in this study. The results are compared with the results of Uslenghi's study.

The time factor $\exp(j\omega t)$ is assumed and suppressed throughout the paper where the angular frequency is ω.

2. THEORY

The soft half-plane is illuminated by the plane wave of

$$u_i = u_0 e^{jk\rho\cos(\phi-\phi_0)} \tag{1}$$

where ϕ_0 is the angle of incidence and u_0 is the complex amplitude. The geometry of the half-plane is given in Fig. 1. The boundary conditions are given as

$$u\big|_s = 0 \tag{2}$$

$$u_1\big|_s = u_2\big|_s \tag{3}$$

and

$$\left.\frac{\partial u_1}{\partial n}\right|_s = \frac{\mu_1}{\mu_2}\left.\frac{\partial u_2}{\partial n}\right|_s \tag{4}$$

where S refers to the surface of the half-plane for the first equation and the boundary of the media for the second and third equations. $\mu_{1,2}$ are the permeability of the mediums.

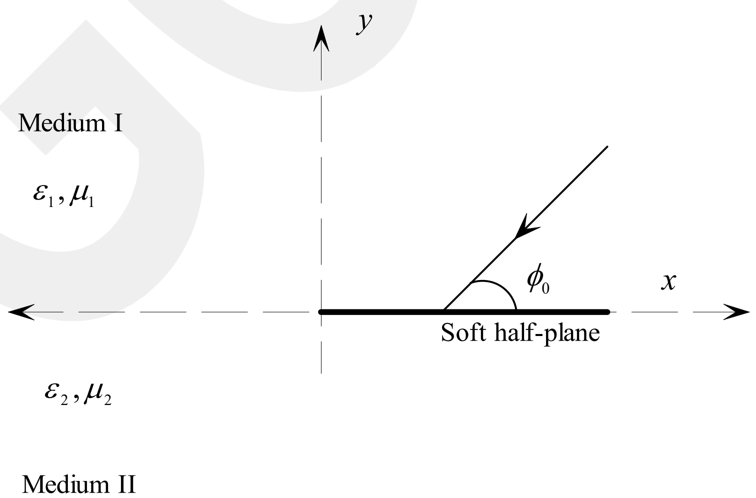

Figure 1:
The geometry of the soft half-plane

The two medium is isorefractive to each other and they have the same wavenumber k but different intrinsic impedance Z. It means that we assume $k_1 = k_2 = k$ where k is equal to $\omega\sqrt{\varepsilon_t \mu_t}$ and $t = 1,2$. The intrinsic impedance Z can be written as

$$Z_t = \sqrt{\frac{\mu_t}{\varepsilon_t}} \tag{5}$$

where $t = 1,2$. According to the method, scattered fields can be written in terms of the combination of scattered fields by soft and hard surfaces with the appropriate multiplication terms. The scattered fields in the first and second medium can be expressed as

$$u_{s1} = \alpha u_{s1}^{s} + \beta u_{s1}^{h} \tag{6}$$

and

$$u_{s2} = \beta\left(u_{s1}^{s} + u_{s2}^{h}\right) \tag{7}$$

where α and β coefficients are dependent to the η term which is the radio of the permittivity of the second medium to first medium and equal to $\varepsilon_2 / \varepsilon_1$. The coefficients are given as

$$\alpha = \frac{\eta}{1+\eta} \tag{8}$$

and

$$\beta = \frac{1}{1+\eta} \tag{9}$$

from Umul and Yalçın (2010). The general solution of the Helmholtz wave equation for the first and second medium can be written as

$$u_{s1} = J_\nu(k\rho)\left[A_\nu \sin(\nu\phi) + B_\nu \cos(\nu\phi)\right] \tag{10}$$

$$u_{s2} = J_\zeta(k\rho)\left[A_\zeta \sin(\zeta\phi) + B_\zeta \cos(\zeta\phi)\right] \tag{11}$$

for this case because of the isorefractive property of the mediums the wavenumbers are taken as the same. ν and ζ are the separation constants. Hence, the application of the soft boundary condition which is given in Eq. (2) on the surface of the half-plane leads to expressions

$$u_{s1} = A_\nu J_\nu(k\rho)\sin(\nu n) \tag{12}$$

$$u_{s2} = \frac{A_\zeta}{\cos(\zeta 2\pi)} J_\zeta(k\rho)\sin(\zeta(\phi - 2\pi)) \tag{13}$$

where the surfaces at $\phi = 0$ and $\phi = 2\pi$. The scattered fields are the composed of the soft and hard subfields so the soft and hard boundary conditions are applied to the fields for $\phi = \pi$. Thus, separation constants are obtained for the scattered fields u_{s1} and u_{s2}. For the u_{s1}^{s} separation constant is obtained as $\nu = n$ and for the u_{s1}^{h} separation constant which is represented by ν_n is found as $2n + 1/2$. The same procedure is also valid for the scattered field of second medium so ζ is equal to n, ν_{n2} is equal to $2n + 1/2$ for the u_{s2}^{s} and u_{s2}^{h} respectively. Here n is an integer. Thus, the fields can be determined as

$$u_{s1} = \alpha A_n J_n(k\rho)\sin(n\phi) + \beta A_{\nu_n} J_{\nu_n}(k\rho)\sin(\nu_n\phi) \tag{14}$$

$$u_{s2} = \beta\left(\frac{A_n}{\cos(n2\pi)} J_n(k\rho)\sin(n(\phi - 2\pi)) + \frac{A_{\nu_{n2}}}{\cos(\nu_{n2})} J_{\nu_n}(k\rho)\sin(\nu_{n2}(\phi - 2\pi))\right) \tag{15}$$

Using the Eq. (6), (7), (12) and Eq. (13). The coefficients, A_n, A_{ν_n} of Eq. (14) and A_n, $A_{\nu_{n2}}$ of Eq. (15) can be determined according to the expressions of

$$\begin{aligned} u_1 = 2\Bigg\{ & \alpha 2u_0 \sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} \sin(n\phi)\sin(n\phi_0) \\ & + \beta u_0 \sum_{n=0}^{\infty} \chi_n J_n(k\rho) e^{jn\frac{\pi}{2}} \cos(n\phi)\cos(n\phi_0) \Bigg\} \end{aligned} \tag{16}$$

and

$$\begin{aligned} u_2 = 2\beta\Bigg\{ & 2u_0 \sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} \sin(n(\phi - 2\pi))\sin(n\phi_0) \\ & + u_0 \sum_{n=0}^{\infty} \chi_n J_{\nu_{n2}}(k\rho) e^{j\nu_{n2}\frac{\pi}{2}} \cos(n(\phi - 2\pi))\cos(n\phi_0) \Bigg\} \end{aligned} \tag{17}$$

where the equations are obtained taking into account the whole plane scattered fields in Umul and Yalçın (2010). The coefficient χ_n is equal to 2 when $n = 0$ and otherwise is 4. Also the angle of refraction is equal to the angle of incidence for the isorefractive medium for this reason ϕ_0 represents the refraction angle in the scattered field of second medium which is given in Eq. (17). The comparison between the Eq. (14), (15) and Eq. (16), (17) gives the related constants. Keep in mind that the reflected and refracted waves do not attenuate so the GO waves have the same amplitude values. As a result scattered fields of soft half plane at the interface of two isorefractive media are obtained as

$$\begin{aligned} u_{s1} = 2\Bigg\{ & \alpha 2u_0 \sum_{n=1}^{\infty} J_n(k\rho) e^{jn\frac{\pi}{2}} \sin(n\phi)\sin(n\phi_0) \\ & + \beta u_0 \sum_{n=0}^{\infty} \chi_n J_{\nu_n}(k\rho) e^{j\nu_n\frac{\pi}{2}} \sin(\nu_n\phi)\sin(\nu_n\phi_0) \Bigg\} \end{aligned} \tag{18}$$

and

$$u_{s2} = 2\beta\left\{2u_0\sum_{n=1}^{\infty} J_n(k\rho)e^{jn\frac{\pi}{2}}\sin(n\phi)\sin(n\phi_0)\right.$$
$$\left.+u_0\sum_{n=0}^{\infty}\chi_n J_{\nu_{n2}}(k\rho)e^{j\nu_{n2}\frac{\pi}{2}}\cos(\nu_{n2})\sin(\nu_{n2}\phi_0)\right\} \quad (19)$$

The total scattered field can be obtained by the addition of Eq. (18) and Eq. (19). The field expressions of Uslenghi for the E-polarization case are given as

$$E_{1z}^{a} = R_e\left(e^{jk\rho\cos(\phi+\phi_0)} - e^{jk\rho\cos(\phi-\phi_0)}\right) \quad (20)$$

$$E_{1z}^{b} = (1+R_e)[G(\phi-\phi_0) - G(\phi+\phi_0)] \quad (21)$$

and

$$E_{1z} = E_{1z}^{a} + E_{1z}^{b} \quad (22)$$

where

$$G(\phi\mp\phi_0) = \frac{e^{j\frac{\pi}{4}}}{\sqrt{\pi}} F\left(-\sqrt{2k\rho}\cos\frac{\phi\mp\phi_0}{2}\right)e^{jk\rho\cos(\phi\mp\phi_0)} \quad (23)$$

where $F(x)$ is the Fresnel function. Also the scattered field of second medium is given as

$$E_{2z} = T_e[G(\phi-\phi_0) - G(\phi+\phi_0)] \quad (24)$$

where the terms R_e and T_e represent the reflection and diffraction coefficients. Total scattered field is obtained by addition of Eq. (22) and Eq. (24). The expressions of the reflection and transmission coefficients are given as

$$R_e = \frac{Z_2 - Z_1}{Z_2 + Z_1} \quad (25)$$

and

$$T_e = \frac{2Z_2}{Z_2 + Z_1} \quad (26)$$

where $Z_{1,2}$ represents the impedance of the mediums and equal to $\sqrt{\mu_{1,2}/\varepsilon_{1,2}}$. The expressions from Eq. (20) to Eq. (26) are taken from the study of Uslenghi which is given in Uslenghi (2013).

The total GO fields can be written as

$$u_{GO1} = e^{jk\rho\cos(\phi-\phi_0)} - e^{jk\rho\cos(\phi+\phi_0)}[U(-\xi_+) + RU(\xi_+)] \quad (27)$$

and

$$u_{GO2} = Te^{jk\rho\cos(\phi-\phi_0)}U(-\xi_-) \quad (28)$$

where $U(x)$ is the unit step function and describes the borders of fields in the media. The detour parameter of the unit step function is written as

$$\xi_{\pm} = -\sqrt{2k\rho}\cos\frac{\phi \pm \phi_0}{2} \tag{29}$$

which is the square root of the two ray path and ρ is the distance between the origin and observation point. R and T are the reflection and transmission coefficients which are given in Umul and Yalçın (2010). The diffracted fields are obtained as

$$u_{d1} = u_{s1} - u_{GO1} \tag{30}$$

and

$$u_{d2} = u_{s2} - u_{GO2} \tag{31}$$

Total diffracted field is obtained by the addition of Eq. (30) and Eq. (31). Similarly total GO field is obtained by the addition of Eq. (27) and Eq. (28).

2. NUMERICAL RESULTS

In this analysis part, we will examine the scattered, diffracted and GO fields. The scattered field expressions which are given in Eq. (18) and Eq. (19) are taking into account. In addition, the GO and diffracted fields through Eq. (27) to Eq. (31) will be plotted. The diffracted fields for the different values of η will be plotted. Moreover the obtained result of scattered field and Uslenghi's result will be analyzed numerically.

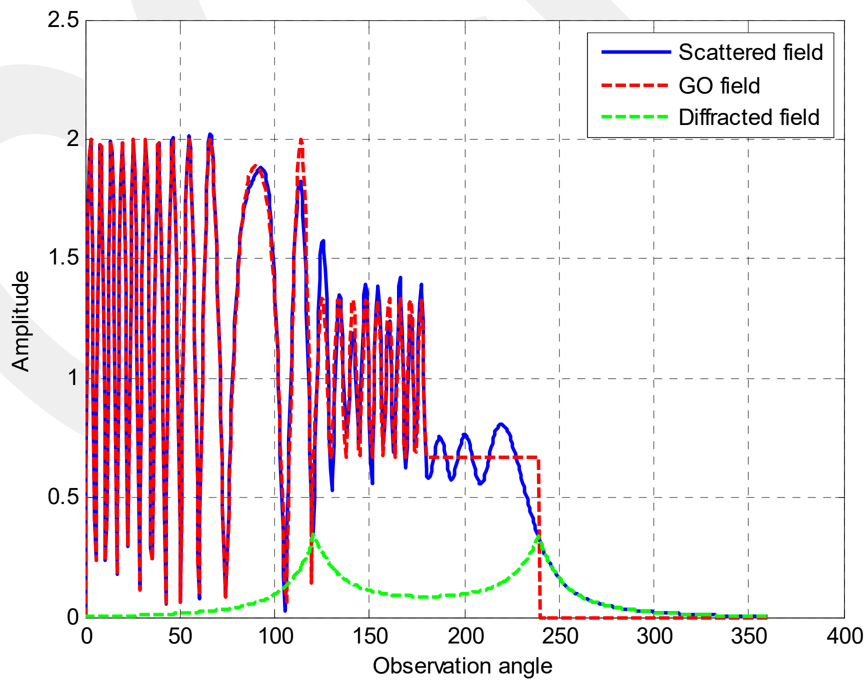

Figure 2:
Total scattered, GO and diffracted fields

In Fig. 2, total scattered, total GO and diffracted fields by the half-plane at the boundary of the isorefractive mediums are shown according to the observation angle. The related parameters are taken related with the wavelength λ. The distance of observation is taken as 6λ. The angle of incidence ϕ_0 is taken as 60˚. The ratio of the permittivity which is represented by η is taken as 2. The diffracted field takes its maximum amplitude values at the reflection 120˚ and shadow boundaries 240˚. In these points diffracted field compensates the discontinuities of the GO fields in the transition regions. The scattered field is continuous at the 180˚ which shows the media boundary.

Figure 3 shows the diffracted field's amplitude variations according to the observation angles. It is clearly seen from the figure that the density of the medium is directly efficient on the amplitude of the diffracted field. η is the ratio of the permittivity of the mediums. Hence, increase of the value of η cause the density increase of medium two according to the medium one. This case less diffracted field amplitude is observed. But keep in mind that mediums are isorefractive so to provide this feature the permeability of the mediums has to be increase or decrease when η changes.

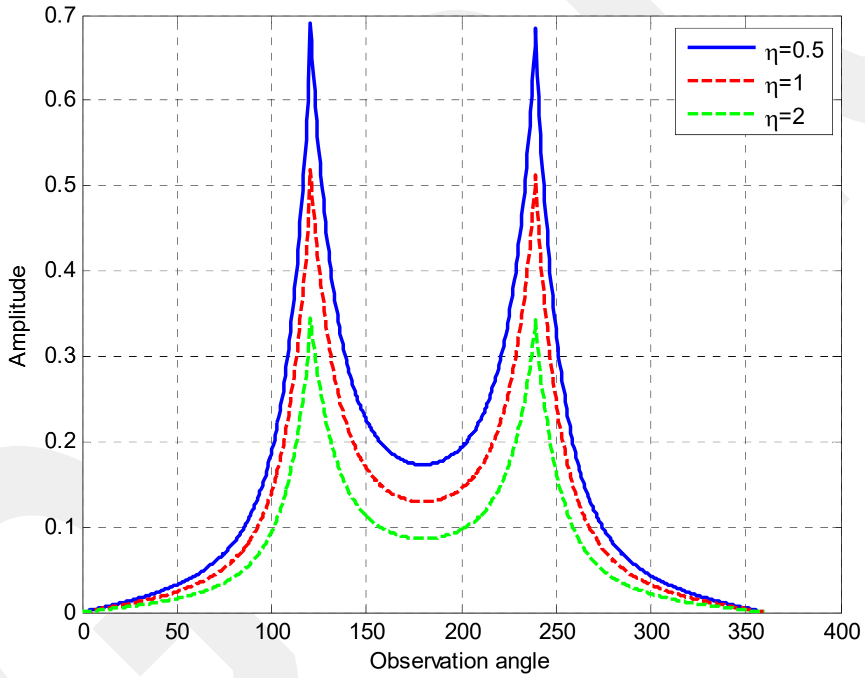

Figure 3:
Diffracted fields for different values of η

Figure 4 shows the scattered fields variation according to the observation angle. Because of the soft boundary conditions the fields have to be taking zero values on the scatterer's surface. The scatterer's surface is at the 0˚ and 360˚. It is seen from Fig. 4 that the scattered fields take zero values in these angles so the boundary conditions are provided. In addition transition between the mediums is continuous at the boundary 180˚. The mentioned observations are valid for both of the solutions. The scattered fields are consistent with each other. Thus, both methods are available for investigation of the scattered field by a half-plane in the boundary of isorefractive mediums.

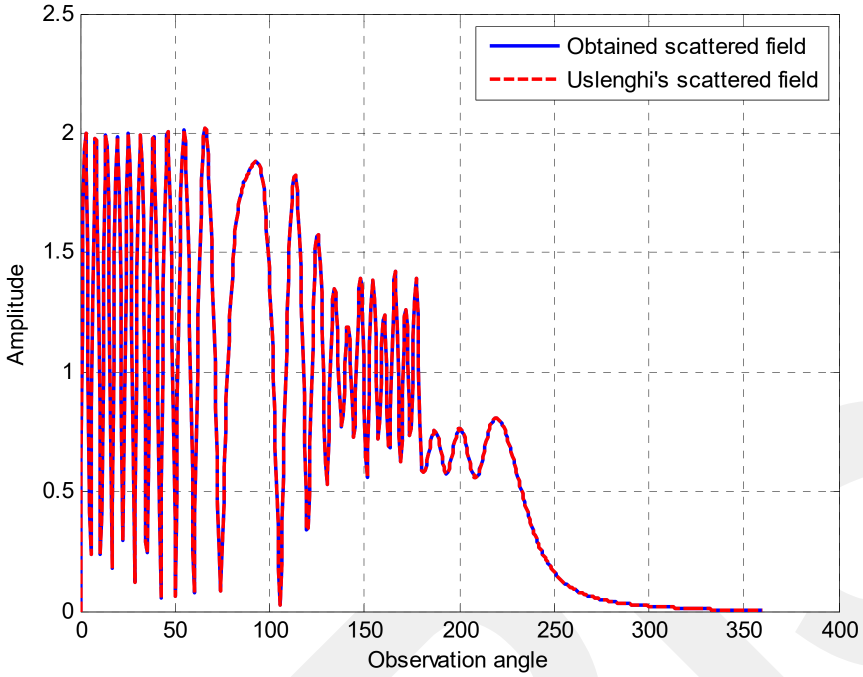

Figure 4:
Total scattered fields

3. CONCLUSION

In this study we derived the scattered, diffracted and GO fields for a half-plane is at the boundary of the isorefractive mediums. The method which is used in this study is based on the series solution. The scattered fields were obtained in terms of the subfields according to the method. The soft boundary condition was taken into account for the half-plane surface. The obtained field expressions were plotted and analyzed numerically. It is observed from plots that the boundary conditions were satisfied by the derived expressions. The results of Uslenghi's similar study were compared by the results of the method used in this study. It is seen that the results were consistent with each other.

REFERENCES

1. Balanis, C.A. (1989). *Advanced Engineering Electromagnetics*, Wiley.

2. Daniele V.G. and Uslenghi, P.L.E. (1999). Exact geometrical optics scattering from a tri-sector isorefractive wedge structure, *IEEE Transactions on Antennas Propagation,* 47, 764-765.

3. Hamid A.K. and Cooray F.R. (2007). Radiation from an axisymmetric slot on a spheroid coated with an isorefractive material and excited by a delta gap source, *IEEE International Conference on Signal Processing and Communications* (ICSPC 2007), 364-367.

4. Tyo J.S., Buchenauer C.J. And Schoenberg J.S.H. (1998). Use of isorefractive media to improve prompt aperture efficiency in a lens IRA, *IEEE Transactions on Antennas Propagation,* 46, 1114-1115.

5. Umul Y.Z. and Yalçın U. (2010). Scattered fields of conducting half-plane between two dielectric media, *Applied Optics*, 49, 4010-4017.

6. Uslenghi, P.L.E. (2004a). Exact geometrical optics scattering from a tri-sector isorefractive wedge structure, *IEEE Antennas Wireless Propagation Letters,* 3, 94-95.

7. Uslenghi, P.L.E. (2004b). Exact geometrical optics scattering from a right angle isorefractive wedge structure, *IEEE Antennas Wireless Propagation Letters,* 3, 127-128.

8. Uslenghi, P.L.E. (2013). Scattering by a half-plane at the interface between isorefractive media , *IEEE Antennas Wireless Propagation Letters,* 12, 1662-1664.

Received : 12.02.2015
Revised : 26.05.2015
Accepted : 26.05.2015